



MCS CONTENT STANDARDS FOR 4th GRADE MATHEMATICS

Fluency Expectations or Examples of Culminating Standards

- 4.NBT.4 Students fluently add and subtract multi-digit whole numbers using the standard algorithm. This is an end-of-year expectation.

The following Standards have changes from the 2015-16 MS College- and Career-Readiness Standards:

Significant Changes (ex: change in expectations, new Standards, or removed Standards)

4.NF.1
4.NF.4c
4.MD.1

Slight Changes (slight change or clarification in wording)

4.OA.3
4.NBT.4
4.NF.3b
4.MD.7

Throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics Grades K-5 Standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: ***fluently***). With respect to student performance and effective in-class instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend to one or more grades earlier in the standards than the grade when fluency is finally expected.

Wherever the word ***fluently*** appears in a MS CCR content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes, but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn't halting, stumbling, or reversing oneself.

A key aspect of fluency is this sense that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

Operations and Algebraic Thinking

Cluster

Use the four operations with whole numbers to solve problems.

Vocabulary: multiplication/multiply, division/divide, addition, add, subtraction/subtract, equations, unknown, remainders, reasonableness, mental math, estimation, rounding

Standard	Clarifications				
<p>4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</p>	<p><i>A multiplicative comparison</i> is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., “<i>a</i> is <i>n</i> times as much/many as <i>b</i>”). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times that quantity is being multiplied.</p> <p>Here students can build on their extensive work in Grade 3 when they learned that the multiplication symbol “\times” means “groups of” and that expressions such as 5×7 can be translated as “5 groups of 7.” Through intentional tasks and discussions facilitated by the teacher, students begin to connect the idea that if “Sarah has two times as many dolls as Jane,” then Sarah’s dolls can be represented as “two groups” of Jane’s dolls.</p> <p><u>Students should have extensive opportunities to explore modeling multiplicative comparisons using pictures and models before writing symbolic equations to represent those relationships.</u></p> <p><u>Example:</u> Tyler’s brother William is three times older than Tyler. If Tyler is 5 years old, how old is William? Use a picture to model and solve the problem and explain how you used the picture to determine your answer.</p> <p><u>Student:</u> First I drew a box for Tyler and labeled it with a T. Then I drew 3 boxes for Wayne and labeled them with a W. I drew 3 boxes because Wayne is 3 times as old as Tyler. That means he’s like 3 groups of Tyler. Then I put my finger on Tyler’s box and thought to myself, “Five” because the problem said he’s five. So, then I put my finger on each of Wayne’s boxes and counted, “5, 10, 15.” So, Wayne is 15 years old.</p> <div style="display: flex; align-items: center; margin-left: 20px;"> <div style="margin-right: 10px;">T</div> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td style="width: 20px; height: 20px;">5</td></tr> </table> </div> <div style="display: flex; align-items: center; margin-left: 20px; margin-top: 5px;"> <div style="margin-right: 10px;">W</div> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td style="width: 20px; height: 20px;">5</td><td style="width: 20px; height: 20px;">5</td><td style="width: 20px; height: 20px;">5</td></tr> </table> </div> <p><u>Teacher:</u> If you had to write an equation to describe the relationship, what do you think that would look like?</p> <p><u>Student:</u> You could say Wayne is 3 groups of Tyler. In third grade, we used “groups of” to talk about multiplication – like, you can read “$6 = 2 \times 3$” as “6 is 2 groups of 3.” So, I could say Wayne = $3 \times$ Tyler because his age is equal to 3 groups of Tyler’s age.</p> <p><u>Teacher:</u> I noticed that you used initials for the boys in your picture and their names in your equations. Why?</p> <p><u>Student:</u> Oh, yeah, I did. I used T and W in my picture just because it was quicker than writing out their whole names. You can do that in the equation, too – like, you could say $W = 3 \times T$, and it’d be the same idea.</p>	5	5	5	5
5					
5	5	5			

4.OA.2

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.¹

¹See Table 2

Table included at the end of this document for your convenience.

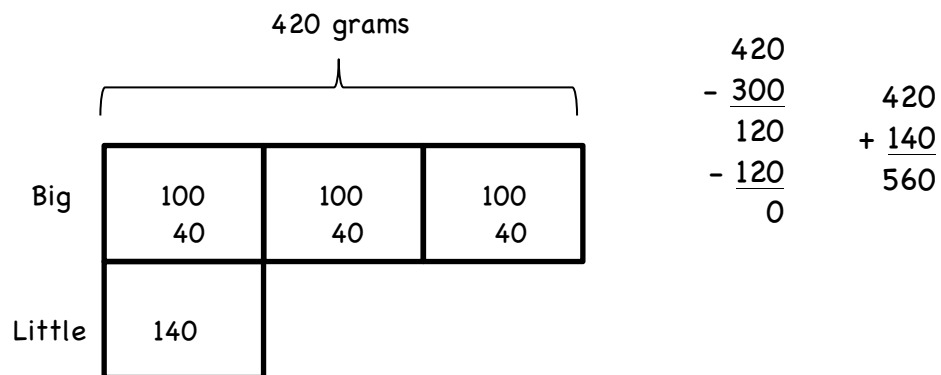
In an additive comparison, the relationship between two quantities is described in terms of addition or subtraction. Ex: “Perry has 3 more pieces of gum than Tim has.” If we knew how many pieces of gum Tim had, we could add 3 to figure out how many pieces Perry has. Or if we knew how many pieces of gum Perry had, we could subtract 3 to determine how many pieces Tim had.

In a multiplicative comparison, the relationship between two quantities is described in terms of multiplication (groups) or division (parts). Ex: “John’s boss makes twice as much money as he does.” If we knew how much money John made, we could multiply that amount by 2 to figure out how much John’s boss made. If we knew how much money his boss made, we could split that amount into 2 equal parts to figure out what John made.

Students should be encouraged to model the relationships in these types of problems first before trying to represent them with symbols or do formal operations. Using pictures and models helps students visualize and make sense the relationships in the problem, which will lead to correct calculations.

Example:

At the zoo, the big penguin eats 3 times as much fish as the little penguin does. The big penguin eats 420 grams of fish each day. How much do the penguins eat altogether in one day? How many more grams of fish does the big penguin eat than the little penguin? Use pictures and words to explain your thinking.



Student: First I drew one box for the little penguin. Then I drew 3 boxes for the big penguin because he eats 3 times as much as the little penguin. I need to figure out how much the little penguin eats. His box is the same size as one of the big penguin’s boxes, so I split 420 into 3 parts. First I put 100 in each box because that was easy. Then I had 120 left to split. I know 3×4 is 12, so 3×40 is 120. So that finished out the 420, and now I can see that the little penguin must ate 140 grams. To figure out how much they ate altogether, I just added up the amounts: $420 + 140 = 560$ grams.

To figure out how much more the big penguin ate than the little penguin, I just looked at my picture and counted it up: 100, 200, 240, 280. So, the big penguin ate 280 grams more than the little penguin did.

4.OA.3

Solve multistep (**two or more operational steps**) word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

The focus in this Standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.

Students should be able to use number sense and estimation strategies to predict “about how much” an answer should be before solving a problem, not just to check the results of their work. By doing so, students can recognize mistakes in calculations along the way and can strengthen their number sense and reasoning skills.

Example:

Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1: Max brought 3 packs of 6 – that’s 18 – and Sarah brought 6 packs of 6, which is 36, 18 and 36, umm... let’s say that’s about 20 and 40, so that’s about 60. We want 300. So $60 + 40$ is 100, and then 200 more would put me at 300. So, I say we need about 240 more bottles. Probably a little bit more than 240 because I rounded up what Max and Sarah had – I said they brought more than they really did, so we’ll need more than 240 in reality.

Student 2: Max brought 3 groups of 6, and Sarah brought 6 groups of 6 – so, if you put them together, that’s really 9 groups of 6, which is 54. So, if they want 300 bottles, 300 minus about 50 would be about 250. So, I think they need to collect about 250 more bottles.

$300 - (36+18) = 246$, so both students’ estimates were reasonable.

Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to:

- Front-end estimation with adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts),
- Rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values),
- Using friendly or compatible numbers such as factors (students seek to fit numbers together - e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000),
- Using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).
- Clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate).

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4.OA.3 (cont'd)

Solve multistep (two or more operational steps) word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Remainders should be put into context for meaningful interpretation.

Ways to address remainders:

- Remainder as a leftover
- Represented as fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer up one more
- Round up/down to the nearest whole number for an approximate result

How one chooses to deal with a remainder depends on the application or context of the story problem. Students should be encouraged to make sense of the any leftover amounts in relation to the problem, rather than being told simply to use a procedure or predetermined representation (Ex 4 R 2) that is not meaningful for them or may not be appropriate for the problem itself.

Students should be encouraged to use units to describe their solutions meaningfully. Responses such as “4 R 2” lack connection to the context of the story. Four *what?* Two *what?* Using units helps students make sense of what they have done and often helps them self-correct calculation errors because they are trying to connect their answer back to the story.

Example: Grandma Mary had 44 flowers. She can fit 6 flowers into a vase.
How many full vases of flowers can Grandma Mary make?

Discussion: $44 \div 6 = 7$ with 2 leftover. In reference to the question that was asked, an appropriate answer would be, “Grandma Mary could fill 7 vases.” The question did not ask us to address any leftover flowers.

Example: Grandma Mary had 44 flowers. She can fit 6 flowers into a vase.
How many full vases of flowers can Grandma Mary make? Will she have any flowers leftover?

Discussion: $44 \div 6 = 7$ with 2 leftover. Now we can address our leftovers meaningfully. An appropriate response to the question asked would be, “Grandma Mary could fill 7 vases. She’d have 2 flowers leftover.”

Example: Grandma Mary had 44 flowers. She can fit 6 flowers into a vase.
How many vases will she need to hold all of her flowers?

Discussion: $44 \div 6 = 7$ with 2 leftover. Here we have to think. She can fill 7 full vases, but she has 2 flowers leftover that will also need a vase. So, an appropriate response to this question would be, “She will need 8 vases to hold all of her flowers.”

The purpose of these examples regarding division and remainders is to illustrate how important it is for students to read the problem carefully and to ensure that they are answering the question that is being asked. Students need extensive experience in exploring different scenarios and in reasoning through how to interpret the result of a calculation in reference to the context of a word problem or real-world scenario.

Cluster**Gain familiarity with factors and multiples.**

Vocabulary: multiplication/multiply, division/divide, factor pairs, factor, multiple, prime, composite

4.OA.4

Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

A prime number has exactly two whole number factors: the number 1 and itself. (There are no other two whole numbers you can multiply together and get that number, besides 1 and the number itself.)

Example: The only two whole numbers that you can multiply to get 17 are 1 and 17. There are no other whole numbers we could multiply together to get 17. So, 17 is a prime number.

A composite number has more than two whole number factors, *including 1 and itself*.

Example: We can multiply 1 and 24 together and get 24. But we can also multiply 2 and 12, 3 and 8, or 4 and 6 to get 24. So 24 is a composite number.

A common misconception is that the number 1 is prime, when in fact; it is not considered to be either prime or composite. Another misconception is that all prime numbers are odd numbers. This is not true: 2 is an even number, and its only whole number factors are 1 and 2 (itself). However, 2 is the *only* even prime number.

A good task for students to investigate prime and composite number is to use square tiles or graph paper to see how many rectangles (or rectangular arrays) they can build from a certain number of square units. Prime numbers will only have two formations because they only have two whole number factors. (Ex: The only rectangles that can be formed with an area of 17 square units have dimensions of 1×17 and 17×1 .) However, a rectangle with an area of 24 square units can be built with multiple dimensions because it has multiple whole number factors that can be used as side lengths.

Students should understand the process of finding factor pairs so they can do this for any number 1 - 100. Teachers can use discussion prompts to help students find organizational strategies that lend themselves to efficiency (ex: looking for pairs or ordering the factors until they “repeat”), rather than “just guessing.”

Examples:

Find the factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.

Find the factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Student: I started at 1 and thought through my multiplication facts: 1×24 , 2×12 , 3×8 , 4×6 , 5 – nope, 6×4 . I’ve already got 4. 7 – nope. 8×3 – I’ve got 3, too. The factors are repeating. I think I’ve got them all.

Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted – e.g., 5, 10, 15, 20. (There are 4 fives in 20.)

Cluster**Generate and analyze patterns.**

Vocabulary: pattern (number or shape), pattern rule

4.OA.5

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations.

Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

Example:

Pattern	Rule	Other Patterns We Notice
3, 8, 13, 18, 23, 28, ...	Start with 3, add 5 each time	There is a repeating pattern in the ones place. The digits repeat 3, 8, 3, 8, 3, 8...
5, 10, 15, 20, 25, 30, 35 ...	Start with 5, add 5 each time	There is a repeating pattern in the ones place. The digits repeat 5, 0, 5, 0, 5, 0. I also notice that there is a repeating pattern in the tens place. Starting at 10, the digit in the tens place stays the same two times and then goes up by 1: 1, 1, 2, 2, 3, 3

It is extremely important for tasks and discussions to have opportunities to follow up with why we think these patterns might be occurring. Often they are related to known number relationships based on our Base 10 system. Investigating “why” helps students to realize that there is structure underlying mathematics and that they can figure out thought-provoking questions. This develops their content knowledge, critical thinking skills, and confidence.

Students should also explore, describe, and predict next numbers for patterns in which the rule is more complex than just adding the same amount each time.

Ex: 1, 2, 4, 7, 11, 16...
 1, 1, 2, 3, 5, 8, 13, 21, 35...
 1, 4, 9, 16, 25, 36...
 2, 4, 8, 16, 32, 64...

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4.OA.5 (cont'd)

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

Example:

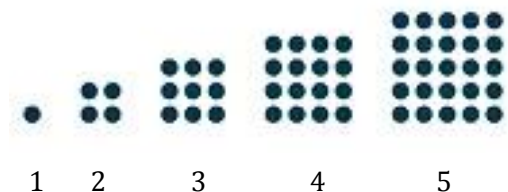
Rule: Create a pattern that starts at 1 and then multiplies each number by 3. Stop when you have 6 numbers. Write down 3 things that you notice.

Student: 1, 3, 9, 27, 81, 243.

- (1) All of the numbers are odd.
- (2) If you add up the digits for each number, they add up to 9.
- (3) The numbers grow really quickly. The difference between 3 and 1 is only 2, and the difference between 9 and 3 is only 6. But the difference between 243 and 81 is a lot more.

Example:

Look at the pattern below. What do you notice about how the pattern? Can you predict how many dots will be at the 6th stage? At the 10th stage?



Example:

Look at the pattern below. What do you notice about the pattern? What shape do you think will be in the 15th place in the pattern? What about the 100th shape?



Number and Operations in Base Ten²

² Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000

Cluster

Generalize place value understanding for multi-digit whole numbers.

Vocabulary: place value, greater than, less than, equal to, $<$, $>$, $=$, comparisons/compare, round

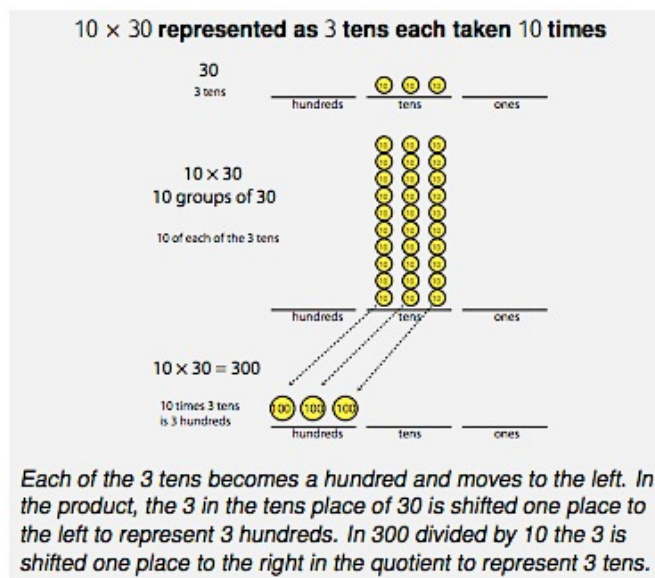
4.NBT.1

Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.

This Standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this Standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with.

This Standard is connected to **4.OA.1** as well as students' prior work with modeling numbers with manipulatives. Base 10 Blocks can be used to model numbers and to discuss why 100 represents "ten times as much" as a 10. Teachers can use discussion prompts to help students use one flat to model a value of 100 and a long to model a value of 1 ten. Just as one flat is worth "ten groups of longs," one hundred is worth "ten groups of ten" (or represents "ten times as many" as one ten.)



In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. Because of this, multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left. (*Progressions for the CCSSM; Number and Operation in Base Ten*, CCSS Writing Team, March 2015, p. 13)

Example: Let's look at the numbers 342 and 324. Is the 2 in 342 the same or different than the 2 in 324? Use the Base 10 Blocks to model 342 and 324. Explain how you used the model to think about the question.

4.NBT.2

Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.

This Standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is $285 = 200 + 80 + 5$. Written form or number name is two hundred eighty-five. *However*, students should also have opportunities to explore the idea that 285 could also be 28 tens and 5 ones or 1 hundred, 18 tens, and 5 ones. Base 10 Blocks work well in such explorations.

Example: How many ways can you find to represent the number 243 using the Base 10 Blocks? Use the chart below to organize your results. What patterns do you notice?

243		
flats	longs	units
2	4	3
1	14	3
0	24	3
2	3	13
2	2	23

To read numerals between 1,000 and 1,000,000, students need to understand the role of commas. [Commas are a notational convention that helps us to recognize and name numbers with many digits.] Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (thousand, million, billion, trillion, etc.). Thus, 457,000 is read “four hundred fifty seven *thousand*” [because the last non-zero digit is in the *thousands* place.] The same methods students used for comparing and rounding numbers in previous grades apply to these numbers, because of the uniformity of the base-ten system. (*Progressions for the CCSSM; Number and Operation in Base Ten*, CCSS Writing Team, March 2015, page 13)

4.NBT.3

Use place value understanding to round multi-digit whole numbers to any place.

This Standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

Example: Jacob and Dylan used rounding to estimate the sum of 171 and 82. Jacob rounded the numbers to the nearest ten and used $170 + 80$ to estimate his answer. Dylan rounded the numbers to the nearest 100 and used 200 and 100 to estimate his answer.

What is the actual sum of 171 and 82? How did the boys’ rounding methods affect their estimates?

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<p>4.NBT.3 (<i>cont'd</i>) Use place value understanding to round multi-digit whole numbers to any place.</p>	<p><u>Examples of students' rounding and estimation strategies in context:</u> On a vacation, your family travels 267 miles on the first day, 194 miles the second day, and 34 miles the third day. How many total miles did they travel?</p> <p><u>Student 1:</u> I looked at 267 and 34 and thought, "That would be about 300. And then 197 is almost 200. So, the answer should be somewhere around 500." When I worked it out, I got 495 miles, and that makes sense.</p> <p><u>Student 2:</u> I looked at the numbers in the problem and thought, "Okay, about 300, about 200, and about 30. So, my answer should be around 530." When I worked it out, I got 801 miles, and I was like, "Whoa! What?" So, I went back and looked, and I'd written 34 down as 340 accidentally. I guess I was in a hurry. But then I fixed it and added them up again and got 495 miles, which is closer to what I thought.</p> <p><u>Student 3:</u> I rounded 267 to 300, 194 to 200, and 34 to 30. Then I put those together and got 530. My estimate is "less than 530" 'cause I rounded up the numbers in the problem kind of a lot. When I added the real numbers from the story, I got 495 miles, which fits what I thought it'd be.</p> <p>The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a reasonable range (between 500 and 550). Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.</p>
<p>Cluster</p>	
<p>Use place value understanding and properties of operations to perform multi-digit arithmetic.</p>	
<p>Vocabulary: add, addend, sum, subtract, difference, equation, strategies, properties (rules about how numbers work), multiple, factor, product, divide, divisor, dividend, quotient, reasonableness, area model, rectangular arrays</p>	
<p>4.NBT.4 <i>Fluently</i> add and subtract multi-digit whole numbers using the standard algorithm.</p> <p>(continued on next page)</p>	<p>Students build on their understanding of addition and subtraction, their use of place value, and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract.</p> <p>This standard refers to fluency, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using a variety strategies such as the distributive property). This is the first grade level in which students are expected to be proficient at using the standard algorithms to add and subtract as an end-of-year objective. However, other previously learned strategies are still acceptable for students to use. The goal is not to take these strategies away from students; the goal is to help them build on their prior knowledge of place value and number relationships and understand how to use the traditional algorithms meaningfully, rather than as a rote procedure that is to be followed.</p>

4.NBT.4 (cont'd)

Fluently add and subtract multi-digit whole numbers using the standard algorithm

The word “algorithm” refers to a procedure or a series of steps that when followed will produce a correct solution. Students should be able to explain how they used the traditional algorithms based on understanding of place value and number. **“Explanations” such as “I followed the steps.” or “More on the floor? Go next door!” are not mathematical explanations and do not demonstrate deep understanding.**

Historically when describing the steps of the standard algorithm, phrases such as, “You can’t take away a bigger number from a smaller number,” are used. This statement should not be used in the classroom because it is inaccurate:

(1) You can subtract a larger amount from a smaller amount. The result is what we call a negative number, often representing the absence of an amount. It is very difficult to get students to “unlearn” this statement when they begin exploring integers (positive and negative numbers) in the sixth grade. It would be better, both for the students’ long-term understanding and for consistency of instruction, if students were not presented with such explanations in elementary mathematics.

(2) The phrase itself demonstrates an inaccurate interpretation of the mathematics at hand. Consider the problem $423 - 156 = \underline{\quad}$. One might hear, “You can’t take away 6 from 3; you can’t take away a bigger number from a smaller number!” as the first step in solving this problem. But this problem isn’t asking us to “remove 6 from 3”; it’s asking us to “remove 156 from 423.” We’re not trying to take away a bigger number from a smaller number at all. Language such as this strips away the relationship of the digits from the overall number and should be avoided.

Students likely will have worked with the traditional algorithms for addition and subtraction in prior grade levels. In Grade 4, the numbers are large enough that the traditional algorithms are often the most efficient algorithms to use. However, attention should be paid to opportunities in which number sense and reasoning strategies are perhaps more appropriate choices.

Example: $4000 - 3880 = \underline{\quad}$

Student: 3880 is close to 4000. I just counted up in my head: 20 more would put me at 3900. And then you need 100 more to get to 4000. So, the difference is 120.

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4.NBT.4 (cont'd)

Fluently add and subtract multi-digit whole numbers using the standard algorithm

Students should be encouraged to use rounding and estimation techniques to predict “about how much” the result should be. This fosters number sense and often helps them self-correct mistakes.

Teacher: writes on the board:

$$\begin{array}{r} 3892 \\ + 1567 \\ \hline \end{array}$$

Let's practice using the traditional algorithm for addition on this one. First, I'd like you to estimate about how much you think the answer will be.

Student: I'd say that first addend is about 4000 and the second addend is, well... it's in between 1000 and 2000. So, I'm going to say it's less than 2000. I'm adding the numbers, so I predict my answer will be less than 6000.

Teacher: Very nice. Now, as you work through this one, I'd like for you to tell me how you're thinking about the problem, okay?

Student: Two ones plus seven ones is nine ones. Nine tens plus six tens is 15 tens. Ten groups of ten make a hundred. So, I'm going to write a 5 down in the tens place and then write a 1 above the hundreds column to show that hundred I just got from the 15 tens. Eight hundreds plus five hundreds plus the extra hundred is 14 hundreds. Ten hundreds make 1000, so I'm going to write a 4 down in the hundreds place and write a 1 above the thousands column to show that new 1000. And then three thousands plus one thousand plus the new thousand (from the hundreds) is five thousand. So, my answer is 5459.

Teacher: That was a very nice explanation; I can see just how you were thinking through that problem. I really like how you explained the ones that you added above the tens column and the hundreds column so that you could remember that that 1 wasn't “just a one”; it represented 1 ten or 1 hundred. Let's look back: Does your answer make sense with what you predicted?

Student: Yeah, it does – 5459 is less than 6000. It's kind of a lot less, but that's probably because 1567 is kind of a lot less than the 2000 I rounded it to. So, I think my answer makes sense.

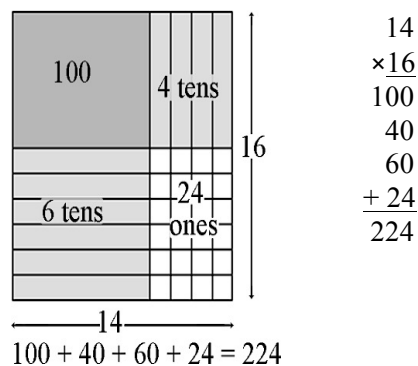
4.NBT.5

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

In Grade 3, students worked with equal group models and array models to explore multiplication within 100. Students should come to Grade 4 with working knowledge of all the products of two one-digit numbers (**3.OA.7**) and the products of one-digit whole numbers and multiples of 10 (**3.NBT.3**).

In Grade 4, we want students to become competent in dealing with larger numbers in multiplication, but we do not want to sacrifice number sense and conceptual understanding in the process. In using Base 10 Blocks, arrays, area models, and alternative algorithms meaningfully, students deepen their understanding of place value and have opportunities to see the usefulness of The Distributive Property of Multiplication in a context. Students should be encouraged to use terms such as “factor” and “product” when discussing multiplication. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. **Fluency with the standard algorithm for multiplication is not expected until the 5th grade.**

Modeling the multiplication of two-digit numbers with Base 10 Blocks can be a powerful visualization for students. Below is a Base 10 Area Model used to find the product of 14×16 . The dimensions of the rectangular area created by the Base 10 Blocks represent the factors 14 and 16. The “product” is the area contained by those dimensions. The Base 10 Area Model is a visual representation of the Partial Products Algorithm, shown next to the model:



The Base 10 Blocks work well for modeling multiplication of reasonably small two-digit numbers, but they may be impractical for modeling larger factors (ex: The time and space it would take to model 85×49 with Base 10 Blocks would likely be more troublesome than helpful for young students.)

After working with physical representations such as the Base 10 Blocks, students can extend their understanding to what is often called a “General Area Model,” (*on the next page*) which maintains the concept of place value without requiring the same level of detail as the Base 10 Block drawings:

(continued on next page)

4.NBT.5 (cont'd)

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Using a General Area Model to model and solve multiplication:



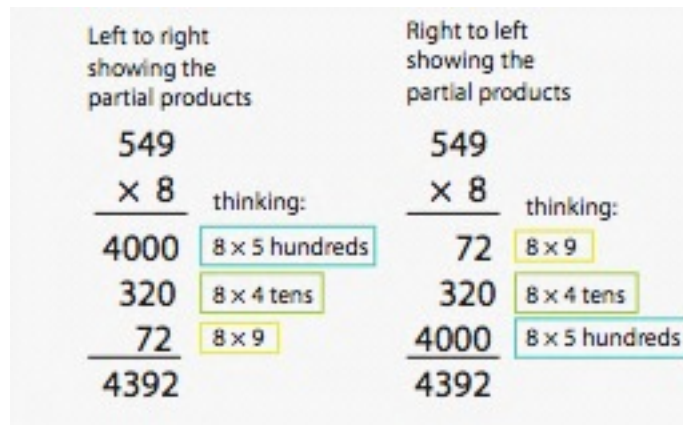
Each part of the region above corresponds to one of the terms in the computation below.

$$\begin{aligned}
 8 \times 549 &= 8 \times (500 + 40 + 9) \\
 &= 8 \times 500 + 8 \times 40 + 8 \times 9.
 \end{aligned}$$

$$\begin{array}{r}
 4000 \\
 320 \\
 + 72 \\
 \hline
 4392
 \end{array}$$

Another part of understanding general base-ten methods for multi-digit multiplication is understanding the usefulness of The Distributive Property of Multiplication over Addition. By decomposing the factors based on place value and applying The Distributive Property of Multiplication over Addition, multiplication of multi-digit numbers can be simplified to a collection of simple or mental math facts.

Students can build on the work with area models to use what is often called the “Partial Products Algorithm”:



Students should not rely on tricks and superficial procedures such as “just add zeros” that do not reflect mathematical understanding. Technically, “add zero” mathematically translates to “+ 0.” When someone says, “To do 6×20 , I do 6×2 and then just add a 0,” what they have actually described in terms of mathematics is “ $6 \times 2 = 12$. $12 + 0 = 12$,” which is clearly not the answer to 6×20 .

(continued on next page)

4.NBT.5 (cont'd)

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

The Base 10 Blocks can be helpful for discussing multiplication with multiples of 10, focusing on place value and meaning. For example, for the problem 4×50 , students could model 4 groups of 5 longs from the Base 10 Blocks to show “4 groups of 50.” Using the Base 10 Blocks and discussion questions, teachers might help students make the connection that “If 4 groups of 5 is 20, I can think of 4 groups of 50 as 4 groups of 5 tens, which would be 20 tens. 20 tens is 200. So, 4×50 is 200.”

From there, students could then move towards mathematical reasoning and fairly quick calculations, without requiring a physical model: $8 \times 500 = 8$ groups of 5 hundreds, which would be 40 hundreds, or 4000.

Another Example:

90 + 4

30	$30 \times 90 =$ $3 \text{ tens} \times 9 \text{ tens} =$ $27 \text{ hundreds} =$ 2700	$30 \times 4 =$ $3 \text{ tens} \times 4 =$ $12 \text{ tens} =$ 120
+	$6 \times 90 =$ $6 \times 9 \text{ tens}$ $54 \text{ tens} =$ 540	$6 \times 4 = 24$

6

Showing the partial products

$$\begin{array}{r} 94 \\ \times 36 \\ \hline 24 \\ 540 \\ 2700 \\ \hline 3384 \end{array}$$

thinking:

- 6×4
- $6 \times 9 \text{ tens}$
- $3 \text{ tens} \times 4$
- $3 \text{ tens} \times 9 \text{ tens}$

Recording the carries below for correct place value placement

$$\begin{array}{r} 94 \\ \times 36 \\ \hline 5 \ 2 \\ 44 \\ 2 \ 1 \\ \hline 720 \\ \hline 3384 \end{array}$$

0 because we are multiplying by 3 tens in this row

(Progressions for the CCSSM; Number and Operation in Base Ten, CCSS Writing Team, March 2015, p.15)

TEACHER NOTE: Students can and should have the opportunity to explore the traditional algorithm for multiplication, as it is often very efficient when dealing with large numbers. But remember that **fluency with the standard algorithm for multiplication is not expected until the 5th grade (5.NBT.5).**

(continued on next page)

4.NBT.5 (cont'd)

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models

Students need extensive experiences to multiply numbers using a variety of strategies that promote number sense and mathematical reasoning. It is important for students to have opportunities to discuss different strategies with their classmates. Over time, discussions can highlight which strategies are more efficient than others, depending on the particular numbers in a problem.

Example: How many total cookies would be in 25 dozen cookies?

Student 1

25 groups of 12 cookies...

$12 \times 12 = 144.$ *12 groups of 12 cookies*

$144 + 144 = 288$ *24 groups of 12 cookies – need 1 more*

$288 + 12 =$

$288 + 10 = 298,$ *and 2 more is 300*

300 cookies

Student 2:

25×12

$= 5 \times 5 \times 12$

$5 \times 12 = 60$

$5 \times 60 = 300$

300 cookies

Student 3:

25×12

$= 25 \times (10 + 2)$

$= 250 + 50$

$= 300$

300 cookies

4.NBT.6

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

In Grade 3, students explored division within 100 in the context of equal groups or equal shares. (See Table 2) Students need opportunities to develop their understandings by using problems in and out of context.

Students can use some of the same models for exploring division as they did for multiplication. Consider:

Division as finding side length

? hundreds + ? tens + ? ones

7

966

$$\begin{array}{r} ??? \\ 7 \overline{)966} \end{array}$$

100 + 30 + 8 = 138

966

-700

266

266

-210

56

56

-56

0

$$\begin{array}{r} 8 \\ 30 \\ 100 \\ \hline 7 \overline{)966} \\ -700 \\ \hline 266 \\ -210 \\ \hline 56 \\ -56 \\ \hline 0 \end{array}$$

(Progressions for the CCSSM; Number and Operation in Base Ten, CCSS Writing Team, April 2012, page 15)

Examples of other strategies for solving division problems are provided below:

Recording division after an underestimate

1655 ÷ 27

Rounding 27 to 30 produces the underestimate 50 at the first step but this method allows the division process to be continued

$$\begin{array}{r} 1 \\ 27 \overline{)1655} \\ -1350 \\ \hline 305 \\ -270 \\ \hline 35 \\ -27 \\ \hline 8 \end{array}$$

“Scaffolding”

$$\begin{array}{r} 8 \\ 30 \\ 100 \\ \hline 7 \overline{)900} \\ -700 \\ \hline 266 \\ -210 \\ \hline 56 \\ -56 \\ \hline 0 \end{array}$$

(Progressions for the CCSSM, Number and Operation in Base Ten, CCSS Writing Team, March 2015, page 17-18)

Number and Operations – Fractions³

³ Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Cluster

Extend understanding of fraction equivalence and ordering.

Vocabulary: equivalent fractions, denominator, numerator, comparison/compare, $<$, $>$, $=$, benchmark fractions

4.NF.1

Recognizing that the value of “ n ” cannot be 0, Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

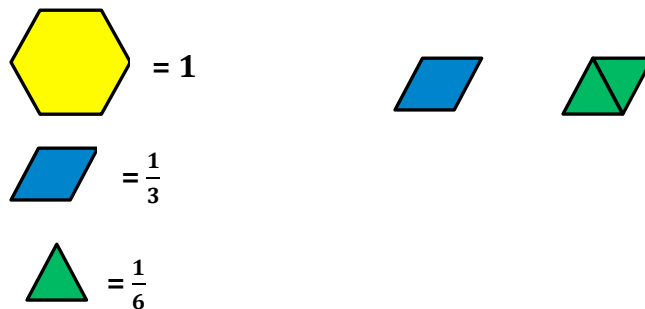
This Standard refers to visual fraction models. This includes area models, number lines, and set models (sets of discrete objects). This Standard extends students’ work in third grade by introducing additional denominators: 5, 10, 12, and 100 to their prior work with denominators of 2, 3, 4, 6, and 8.

Physical manipulatives that work well as visual models for fractions with this range of denominators include Pattern Blocks, Cuisenaire Rods, Fraction Tiles, Fraction Bars, and Base 10 Blocks.

Students have initially explored models for equivalent fractions in Grade 3 (**3.NF.3**).

Example: Do $\frac{1}{3}$ and $\frac{2}{6}$ represent the same amount, or is one more than the other?
Use the pattern blocks to show your thinking.

Student: I’m going to use the yellow hexagon to be my whole.



It takes 3 blue rhombi to make a hexagon, so each rhombus is $\frac{1}{3}$ of the hexagon. It takes 6 green triangles to make the hexagon, so each triangle is $\frac{1}{6}$ of the hexagon; and so 2 triangles would be $\frac{2}{6}$ of the hexagon. 1 rhombus and 2 triangles cover the same space in the hexagon, so $\frac{1}{3}$ and $\frac{2}{6}$ show the same amount, just in different ways.

Teacher: Do you know what we call fractions that represent the same amount?

Student: Yeah, we call them equivalent fractions.

Teacher: That is what we call them. Now, I have a question for you. What would you say to someone who said that $\frac{1}{3}$ and $\frac{4}{6}$ were equivalent fractions because $1 + 3 = 4$, and $3 + 3 = 6$?

(continued on next page)

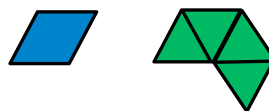
4.NF.1 (cont'd)

Recognizing that the value of “n” cannot be 0, Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Student: That’s not how equivalent fractions work. Equivalent fractions represent the same amount. $\frac{1}{3}$ and $\frac{4}{6}$ aren’t the same amount.

Teacher: Can you use the pattern blocks to show me?

Student: It’s like before. I’m going to use the yellow hexagon as a whole. One blue rhombus is $\frac{1}{3}$ of the hexagon, and one green triangle is $\frac{1}{6}$ of the hexagon.



It takes 2 triangles to make 1 rhombus, so you could also think of one third as two sixths. $\frac{4}{6}$ would be four triangles, and that’s more than one rhombus. They don’t represent the same amount, so $\frac{1}{3}$ and $\frac{4}{6}$ aren’t equivalent fractions.

4.NF.2

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Students need extensive experience in comparing fractional amounts using pictures and models before attempting traditional procedures for creating equivalent fractions or comparing fractional amounts.

Pattern Blocks, Cuisenaire Rods, Fraction Tiles, Fraction Bars, number line models, and area models are all useful representations for modeling fraction amounts and relationships. Sometimes one model lends itself better to a particular fractional amount. (Ex: Some students prefer using Pattern Blocks to model halves, thirds and sixths and Cuisenaire Rods to model fourths, fifths, eighths, and tenths.) Students should be able to choose from a variety of fraction models to decide what works best for a particular scenario – this can only come from working with multiple models over time.

Students’ conceptual understanding and work with visual models for fractions are critical for them to have long-term understanding and success with the traditional method for finding common denominators later.

Similarly, students will not be able to understand and be able to use traditional procedures such as “cross-multiplying” if they do not have a strong conceptual understanding of “how much” fractional amounts represent. Students should have extensive experience using pictures and models, number sense, and benchmark fractions to compare fractional amounts before considering the introduction of a “cross-multiplying” strategy. **“Cross-multiplying” is not necessary to compare fractional amounts, can take more time to carry out, and lends itself more to mistakes and misunderstandings than the meaningful methods upheld within the expectations of this Standard.**

TEACHER NOTE: Students should not be using calculators to deal with fractions at this grade level – not as fractions, and not as fractions converted to decimals.

(continued on next page)

4.NF.2 (cont'd)

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$.

Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Example: (Returning to prior example with student regarding $\frac{1}{3}$ and $\frac{4}{6}$):

Teacher: So, you said that $\frac{1}{3}$ and $\frac{4}{6}$ aren't equivalent fractions. So, one fraction must be bigger or smaller than the other one. What do you think?

Student: Four triangles cover more space than one rhombus, so four sixths is more than one third.

Teacher: Remember when we used our symbols for greater than and less than to compare whole numbers? How could we use one of those symbols to describe what you just said?

Student: writes $\frac{4}{6} > \frac{1}{3}$

Teacher: So, what does that say?

Student: Four sixths is greater than one third.

Teacher: Very nice. Now, you said that $\frac{1}{3}$ and $\frac{2}{6}$ were equivalent fractions. Does $\frac{4}{6}$ have an equivalent fraction?



Student: Four triangles cover the same space as two rhombi. And since each rhombus represents one third, then you could say that *four* sixths is the same as *two* thirds. So, four sixths and two thirds would be equivalent fractions.

Teacher: How could we describe that relationship using symbols?

Student: writes $\frac{4}{6} = \frac{2}{3}$

Teacher: What does that equation tell us? How can we read it in words?

Student: Four sixths is equal to two thirds. That means they're the same amount.

Teacher: Very nice. Are there any other relationships like that in the pattern blocks? Or is it just with the triangles and the rhombi?

Student: No, the triangles and the trapezoid fit together, too.

Teacher: Can you show me?

(continued on next page)

4.NF.2 (cont'd)

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Student: Sure. See, you can make a red trapezoid with three green triangles.



Teacher: I see. So, how could we describe that relationship in terms of fractions?

Student: One trapezoid is half of a hexagon. And I already said how a triangle is a sixth of the hexagon. So, that means that *one* half is equal to *three* sixths.

Teacher: Okay. So, let's write down that relationship using symbols, too.

Student: writes $\frac{1}{2} = \frac{3}{6}$

Example of using a benchmark fraction to compare:

Teacher: Is $\frac{2}{6}$ more than, less than, or the same amount as $\frac{3}{4}$? Explain how you thought about the fractions. Express your answer using words and symbols.

Student: I know $\frac{3}{4}$ is more than one half because half of four is two. So, one half would be two fourths, and I have three fourths, which is more. $\frac{2}{6}$ is less than a half because half of six is three. So, one half would be three sixths, and I only have two sixths, which is less. So, $\frac{2}{6}$ is less than $\frac{3}{4}$, which I would write as $\frac{2}{6} < \frac{3}{4}$.

Example of using a reasoning strategy to compare: Is $\frac{7}{8}$ more than, less than, or the same amount as $\frac{4}{5}$? Explain how you thought about the fractions. Express your answer using words and symbols.

Student: Both fractions are close to one whole. Actually, both of them are just missing one piece to be one whole. But the pieces they're missing aren't the same size. $\frac{7}{8}$ needs $\frac{1}{8}$ to make a whole, and $\frac{4}{5}$ needs $\frac{1}{5}$ to make a whole. Fifths are bigger than eighths because the fewer pieces that you cut the whole into, the bigger the pieces are. So, $\frac{4}{5}$ needs a bigger piece to make one whole than $\frac{7}{8}$ does. $\frac{7}{8}$ is actually closer to being one whole than $\frac{4}{5}$ is. So, $\frac{7}{8}$ is more than $\frac{4}{5}$, which I would write as $\frac{7}{8} > \frac{4}{5}$.

Cluster

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Vocabulary: unit fraction, equivalent fractions, denominator, like/unlike denominators, numerator, mixed numbers,

4.NF.3

Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model (including, but not limited to: concrete models, illustrations, tape diagram, number line, area model, etc.).

Examples: $3/8 = 1/8 + 1/8 + 1/8$;
 $3/8 = 1/8 + 2/8$; $2 \ 1/8 = 1 + 1 + 1/8$
 $= 8/8 + 8/8 + 1/8$

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

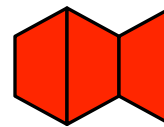
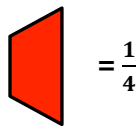
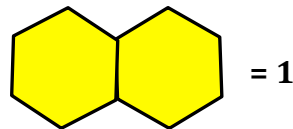
Students first explored unit fractions in Grade 3 (3.NF.1).

Students can use models (ex: Pattern Blocks, Cuisenaire Rods, Fraction Tiles) to visualize that a fraction such as $7/8$ is composed of 7 groups of $1/8$, where $1/8$ represents one of the eight equal parts in the whole.

Students should talk about fractions using pictures, models, and words before using traditional symbolic notation for fractions.

Example: If I use two yellow hexagons joined together to represent 1 whole, then how much would three red trapezoids represent?

Student: Two yellow hexagons together make one whole.



It takes four red trapezoids to cover that whole. So, one trapezoid would be one fourth of the whole, and so three trapezoids would be three groups of one fourth: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ which is three fourths: $\frac{3}{4}$

Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and need to be able to decompose them in more than one way. Students may use visual models to help develop this understanding.

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4.NF.3 (cont'd)

Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model (including, but not limited to: concrete models, illustrations, tape diagram, number line, area model, etc.).

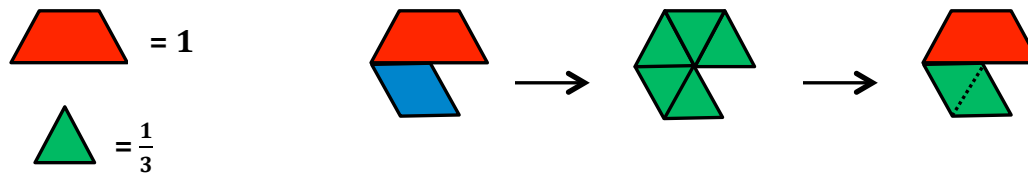
Examples: $3/8 = 1/8 + 1/8 + 1/8$;
 $3/8 = 1/8 + 2/8$; $2\ 1/8 = 1 + 1 + 1/8$
 $= 8/8 + 8/8 + 1/8$

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

(continued on next page)

Example: If a trapezoid represents 1 whole, how much would a trapezoid and a rhombus joined together represent?



Student: It takes three triangles to make a trapezoid, so each triangle would be one third of the trapezoid. It takes five triangles to cover the same space as a trapezoid and a rhombus put together. So, that would be five thirds.

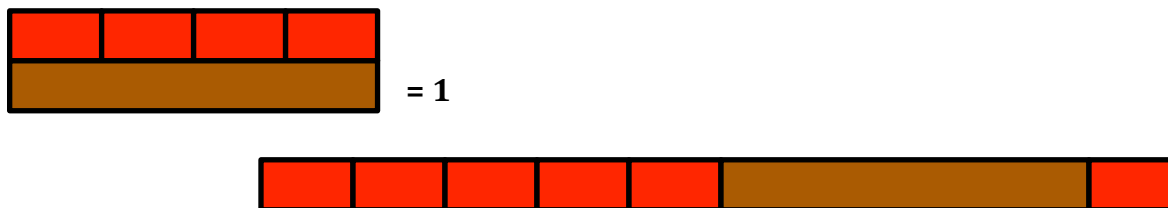
Teacher: I see what you're saying. Five thirds is a good mathematical way to describe that amount. I'm curious – is there another way we might describe it?

Student: Well, when you look at it, you've already got one trapezoid in that trapezoid and rhombus joined together. So that's a whole. Then you just have to figure out how much the rhombus represents. You can cover the rhombus with two triangles; so since each triangle is one third of the whole – the trapezoid – then the rhombus would represent two thirds. So, you could also say that you have one whole and two thirds.

Students need extensive experience in modeling addition and subtraction problems with pictures and models before connecting that work to symbols, equations, or traditional procedures.

Example: Angela is collecting pieces of ribbon to use to make a sewing project with her grandmother. Angela has one piece that is $3/4$ of a yard long, a second piece that is $2/4$ of a yard long, and a third piece that is $1\ 1/4$ yards long. If Angela put the ribbons end to end, how long would they be?

Student: I used the Cuisenaire Rods. I let the brown rod be one whole. It takes four red rods to make a brown rod, so each red rod represents one fourth.



I put three red rods together to show the first piece of ribbon. Three red rods would be three fourths. Then I put two more red rods next to those to show the second piece that was two fourths of a yard. And then I put a brown rod and another red rod next to those to show the last piece of ribbon.

4.NF.3 (cont'd)

Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model (including, but not limited to: concrete models, illustrations, tape diagram, number line, area model, etc.).

Examples: $3/8 = 1/8 + 1/8 + 1/8$;
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c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

(continued on next page)

I wanted to think about it in terms of the red rods, so I lined four red rods up underneath the brown rod so that I could see it better.



Then I counted up how many red rods I had, which was 10. So, the ribbons would be $10/4$ yards long together.

Teacher: I see. I agree with you: they would be $10/4$ yards long together. Mathematically, that's true. Typically, when people buy ribbon or fabric at a store, they ask for "so many" whole yards and maybe a little bit more. Is there a way we could describe your answer in terms of whole yards?

Student: Well, the brown rod is my whole, so you could look at it like that.



If I line up my brown rods under the red rods, I can use two of them. I can't put a third one there, though – that's too long. So, you could say it takes two brown rods and two red rods to show the ribbon.

Teacher: How could we describe the two brown rods and two red rods in relation to the problem?

Student: Two brown rods and two red rods would be two whole yards and two fourths more yards.

Teacher: How could we write a number sentence for this problem?

Student: I put $\frac{3}{4}$, $\frac{2}{4}$, and $1\ \frac{1}{4}$ together and got $2\ \frac{2}{4}$. So, I would write $\frac{3}{4} + \frac{2}{4} + 1\ \frac{1}{4} = 2\ \frac{2}{4}$.

Teacher: Is there anything else that you notice about the problem or your model?

Student: You could look at it as just adding a bunch of fourths together. 1 whole is 4 fourths. So, really, you've got 3 fourths plus 2 fourths plus 4 fourths plus 1 fourth, which is 10 fourths, like I had before.

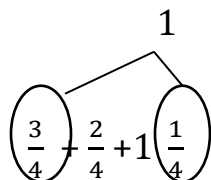
Teacher: Could you write a number sentence to show how you're thinking about that?

Student writes $\frac{3}{4} + \frac{2}{4} + \frac{4}{4} + \frac{1}{4} = \frac{10}{4}$.

4.NF.3 (cont'd)

Example of using number sense/reasoning:

$$\frac{3}{4} + \frac{2}{4} + 1\frac{1}{4} = \underline{\hspace{2cm}}$$



$$= 1 + 1 + \frac{2}{4} = 2\frac{2}{4}$$

TEACHER NOTE: The term “reduce” should be avoided when discussing equivalent fractions, as it often leads to misinterpretation by students. When some students hear, “I reduced $\frac{3}{6}$ to $\frac{1}{2}$,” they often infer that $\frac{1}{2}$ is *smaller than* $\frac{3}{6}$ because “reduce” means “to make smaller.” This error is compounded by the fact that the digits in $\frac{1}{2}$ are of smaller numerical value than the digits in $\frac{3}{6}$. A more accurate (and less confusing) term to use is to “simplify” fractions. Thus we could “simplify” $\frac{3}{6}$ to $\frac{1}{2}$.

4.NF.4

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

- a. Understand a fraction a/b as a multiple of $1/b$.

For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

Students may worry that they have to find and use new methods for modeling multiplication of fractions, but the underlying concepts are the same as with whole numbers. Consider the following Standard from Grade 3: **3.OA.1** – “Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. Just as we can interpret 2×4 as “2 groups of 4,” we can read $2 \times \frac{2}{3}$ as “2 groups of $\frac{2}{3}$.”

It is vital that students have extended experiences with using visual models before moving forward into symbols, equations, or traditional procedures.

4.NF.4a is connected to **4.NF.3** to see examples of how to help students use fraction models to visualize and make sense of this concept.

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4.NF.4 (cont'd)

- b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number.

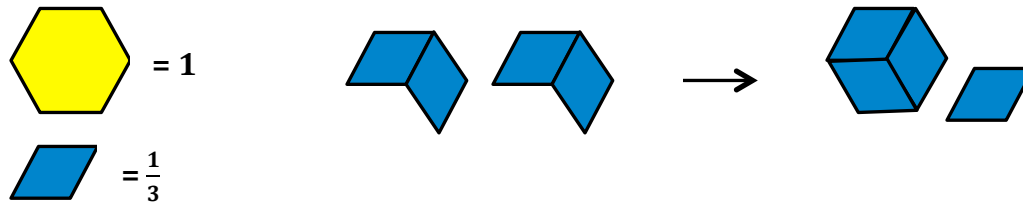
For ex., use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)

- c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

*For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? **Between what two whole numbers do you expect your answer to lie?***

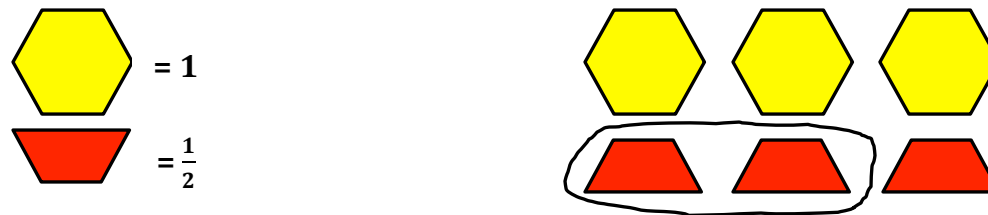
Example: Use the pattern blocks to model and solve $2 \times \frac{2}{3}$. Explain in 2-3 sentences how you used the pattern blocks to think about the problem.

Student: I let the yellow hexagon represent one whole.



It takes three rhombi to make a hexagon, so one rhombus would be $\frac{1}{3}$ of the hexagon, and two rhombi would be $\frac{2}{3}$ of the hexagon. $2 \times \frac{2}{3}$ means two groups of $\frac{2}{3}$, so I made two groups of two rhombi. That's four rhombi altogether, so the answer is $\frac{4}{3}$. Or, you could rearrange the rhombi to show that they cover the same area as a hexagon with another rhombus leftover. So that would be one whole and one third more or $1 \frac{1}{3}$.

Example: Jackie is making Kool-Aid for her Girl Scout Troup. She has three packages of Kool-Aid, and each package needs $1 \frac{1}{2}$ cups of sugar. How many cups of sugar does Jackie need to make all of the Kool-Aid?



Student: $1 \frac{1}{2}$ is between 1 and 2, so 3 groups of $1 \frac{1}{2}$ should be between 3 and 6. I used pattern blocks to represent cups of sugar and made 3 groups of $1 \frac{1}{2}$ – one for each pack of Kool-Aid. Then I looked at how much I had altogether. I put two trapezoids together because that's like another hexagon. So, now I have 1, 2, 3, 4, wholes and a trapezoid leftover – that's a half. So, Jackie needs $4 \frac{1}{2}$ cups of sugar. $4 \frac{1}{2}$ is between 3 and 6, so I think my answer makes sense.

Teacher: What number sentence do you think best describes this problem?

Student: I needed to find out how much 3 groups of $1 \frac{1}{2}$ was, so I would write $3 \times 1 \frac{1}{2} = 4 \frac{1}{2}$.

Cluster**Understand decimal notation for fractions, and compare decimal fractions.**

Vocabulary: fraction, numerator, denominator, equivalent fractions, tenths, hundredths, comparison/compare, <, >, =

4.NF.5

Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.⁴

For example, express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.

⁴ Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

This Standard builds on students' prior experiences with modeling our place value system using Base 10 Blocks as well as using visual representations to recognize and explain equivalent fractions.

Work with decimals is introduced in Grade 4 (4.NF.6 and 4.NF.7). Working with fractions that have denominators of 10 and 100 open the door for discussions of tenths and hundredths of a whole, paving the way for students' understanding of place value to the right of the decimal point in our traditional Base 10 number system.

Base 10 flats and decimal grids (10×10 grids that resemble Base 10 flats) provide important visual models for students to explore these concepts and relationships. Student experiences should focus on working with models that support conceptual understanding rather than algorithms and procedures at this grade level. (See footnote⁴ below the Standard.)

Example: Let's let the Base 10 flat represent one whole.
How many longs does it take to make one flat?

Student: It takes ten longs to make a flat.

Teacher: Does each long represent an equal part of the flat?

Student: Yes.

Teacher: So, could we describe a long as a fraction of a flat?

Student: It takes 10 longs to make a flat, so 1 flat would be one tenth of a flat.

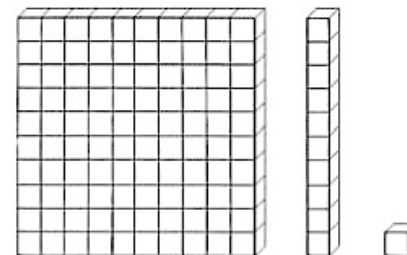
Teacher: How would you write that using our fraction notation?

Student: writes $\frac{1}{10}$

Teacher: Okay. What about our Base 10 unit? How many units does it take to make a flat?

Student: A hundred.

Teacher: So, could we describe a unit as a fraction of a flat?



(continued on next page)

4.NF.5

Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.⁴

For example, express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.

⁴ Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

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Student: If it takes a hundred units to make a flat, I guess that would make a unit a hundredth?

Teacher: That's right. So, how could we write "one hundredth" using our fraction notation?

Student: writes $\frac{1}{100}$ That's a pretty big number in the denominator. That's the biggest one we've had so far.

Teacher: It is. Some people might look at the 100 in the denominator and think that since 100 is a big number, $\frac{1}{100}$ would be a big number. What do you think?

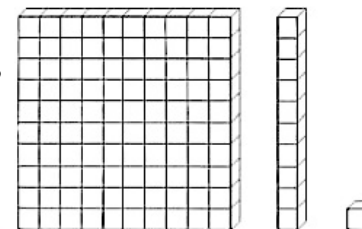
Student: 100 is a big number, but that's not how fractions work. The denominator tells you how many equal parts the whole is split into. A whole split into 100 parts would actually give you really tiny parts.

Teacher: So, would $\frac{1}{10}$ be bigger or smaller than $\frac{1}{100}$?

Student: One tenth would be bigger because it would be one of ten equal parts in the whole. If the whole is split into 10 parts, those parts still probably aren't very big, but they'd be a lot bigger than if you cut the whole into 100 parts.

Teacher: How could we use Base 10 Blocks to show what you just described?

Student: It takes 10 longs to make a flat and 100 units to make a flat. So, a unit – one hundredth – is a lot smaller than a long – one tenth – because it takes a lot more units to cover the same space as ten longs do.



Teacher: You said one long represents one tenth of the flat. If I wanted to describe the long in terms of hundredths, how could I do that?

Student: It takes 10 units to make a long, so 1 long is equal to 10 hundredths. You could write that like $\frac{10}{100}$.

Teacher: That is how we would write ten hundredths; good job. Suppose we wanted to use the Base 10 Blocks to model $\frac{2}{10} + \frac{30}{100}$. What could we do?

Student: We could get 2 longs for the two tenths and 30 units for the thirty hundredths. That's a lot of units. But, really, 30 units are the same as 3 longs. So you could probably use those instead, and it would be easier. And then you'd put all the longs together, and you'd have five longs.

Teacher: So, how could we describe the five longs in terms of a fraction here?

Student: If one long is one tenth of the flat, then five longs would be five tenths. So you could write $\frac{5}{10}$.

<p>4.NF.5 (cont'd)</p>	<p><u>Teacher</u>: I see. Is that the only fraction we could write?</p> <p><u>Student</u>: Well, five longs are the same as fifty units. So, if a unit is a hundredth, then you could also say the answer was $\frac{50}{100}$. It's the same amount; I guess it just depends on how you want to describe it.</p>
<p>4.NF.6 Use decimal notation for fractions with denominators 10 or 100.</p> <p><i>For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</i></p>	<p>Decimals are introduced for the first time in Grade 4. Students should have ample opportunities to explore and reason about the idea that a number can be represented <i>as both a fraction and a decimal</i>.</p> <p>“The decimal point is used to signify the location of the ones place, but its location may suggest there should be a “oneths” place to its right in order to create symmetry with respect to the decimal point. However, because one is the basic unit from which the other base ten units are derived, the symmetry occurs instead with respect to the ones place.”</p> <div data-bbox="879 565 1644 812" data-label="Diagram"> </div> <p>(Progressions for the CCSSM; Number and Operation in Base Ten, CCSS Writing Team, March 2015, p. 14)</p> <p>Exploring decimal values is an extension of the place value concepts students have already explored with whole numbers. Just as 15 is understood as both “1 ten and 5 ones” <i>and</i> as “15 ones,” 0.15 can be described as “1 tenth and 5 hundredths” <i>and</i> as “15 hundredths.”</p> <p>Using the Base 10 Blocks and/or decimal grids and in-class discussions (see example from 4.NF.5) can help students develop a deeper understanding of decimal values <i>and</i> the relationship between fractions and decimals. It is important for students to understand (for example) that $\frac{5}{10}$ and .5 are describing the same amount, just in different ways.</p>
<p>4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.</p>	<p>Similar to their work with comparing fractions in 4.NF.2, students should understand that comparisons of decimals are only valid when they refer to the same whole. (For example, saying that one tenth is less than one hundredth because one tenth of a centimeter (i.e., a millimeter) is smaller than one hundredth of a kilometer (i.e., a dekameter) is not a valid argument.</p> <p>Visual models that are useful for these comparisons include Base 10 Blocks, decimal grids, and metric references (see 4.MD.1).</p>

Measurement and Data

Cluster

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Vocabulary: measure, metric, customary, convert/conversion, relative size, liquid volume, mass, length, distance, kilometer (km), meter (m), centimeter (cm), millimeter (mm), kilogram (kg), gram (g), milligram (mg), liter (l) milliliter (ml), cubic centimeter, inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), time, hour, minute, second, equivalent, operations, add, subtract, multiply, divide, fractions, decimals, area, perimeter

4.MD.1

Know relative sizes of measurement units within one system of units including km, m, cm, **mm**; kg, g, **mg**; lb, oz; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.

For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

(continued on next page)

In Grade 3, students should have worked with the following measurement units:

Time: hours, minutes

Mass: grams, kilograms

Volume: liters

Area: square cm, square m, square inches, square feet

Length: centimeters, inches, feet

TEACHER NOTE: One milliliter is equal to 1 cubic centimeter. The sides of a Base 10 unit measure 1 cm in length, so a Base 10 unit represents 1 cubic centimeter of volume and would hold 1 milliliter of liquid. When people in the medical profession refer to “cc”s, they are referring to “cubic centimeters.”

Class discussions should facilitate the development of a set of reliable “personal benchmarks” to use as references for measurements within both the metric and customary measurement systems. When students have meaningful references, they are more likely to make sense of measurement relationships.

“Reliable” benchmarks refer to using a reference that is not subjective or dependent on the individual. For example, it is often said that an inch is approximately the distance between the first and second knuckles on your index finger. But children’s hands are very small (and still growing), so this is not a very reliable reference. Personal benchmarks are not expected to be exact, but they should be reasonable estimates. As such, benchmarks for measurement should always be accompanied by the term “about” or “approximately” because they are not exact measurements.

Examples of reliable personal benchmarks for measurement include

- About 1 meter – the distance from the floor to the doorknob on a door
- About 1 decimeter – the length of the longest side of a Base 10 long
- About 1 centimeter – the length of the edge of a Base 10 unit or the width of a standard pencil eraser
- About 1 millimeter – the thickness of a dime
- About 1 gram – the mass of a large paperclip
- About 1 cubic centimeter – (the volume of) a Base 10 unit
- About 1 milliliter – (the volume of) a Base 10 unit

4.MD.1 (cont'd)

Know relative sizes of measurement units within one system of units including km, m, cm, mm; kg, g, mg; lb, oz; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.

For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2,24), (3, 36),...

Students did not convert measurements in Grade 3. In Grade 4, the focus is on expressing measurement in a larger unit in terms of a smaller unit – i.e., “going from larger to smaller units.” This allows students to build on their work with multiplying whole numbers. **Since decimals are introduced for the first time in Grade 4 (4.NF.5, 4.NF.6, 4.NF.7), “going from smaller to larger units” is deferred until Grade 5 to allow students time to build a strong foundation with decimals before using them in this way.**

The intent of this Standard is not to carry out formal conversions. The intent is to provide students with opportunities to explore measurement by using two-column tables to look for patterns and relationships (see **4.OA.5**). These explorations provide opportunities to connect back to **4.OA.1** and discuss the idea of “times as many.” (Ex: “It takes three times as many feet to cover the same distance as it does in yards.”) Exploring relationships within the metric system also provides opportunities to discuss our Base 10 place value system (**4.NBT.1**) in a real world context.

Example: There are 3 feet in 1 yard. I’ve started a table to help us look at this relationship. How can we use the information in the table to fill out the rest of the table? Is there a “rule” we can use to figure out how many feet are in a certain number of yards?

Yards	Feet
1	3
2	6
3	9
4	
5	
10	
20	
100	

TEACHER NOTE: These Standards do not differentiate between weight and mass. Technically, mass is the amount of matter in an object. Weight is the force exerted on the body by gravity. On the earth’s surface, the distinction is not important (on the moon, an object would have the same mass, would weigh less due to the lower gravity). (*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, p. 2)

4.MD.2

Use the four operations to solve word problems involving

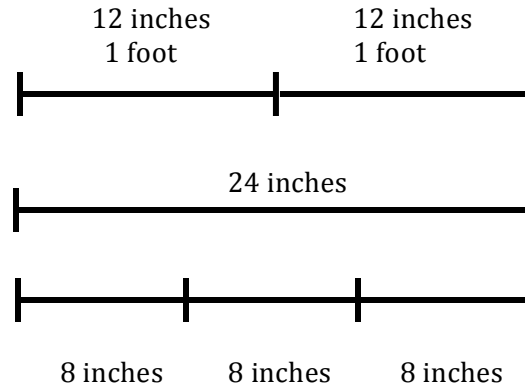
- intervals of time,
- money
- distances
- liquid volumes
- masses of objects

including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Students should have ample opportunities to create pictures, charts, and models (ex: number line) to represent the quantities in story problems. Using these representations as part of solving the problem promotes conceptual understanding, number sense, and overall problem-solving skills.

Example: Susan has 2 feet of ribbon. She wants to use all of her ribbon to make friendship bracelets for her 3 friends. How many inches of ribbon should she cut for each friendship bracelet?

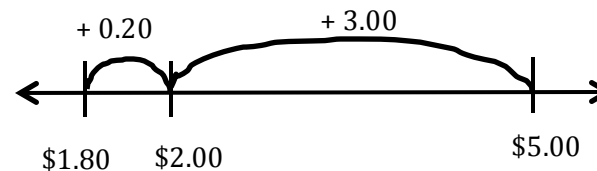
Student: There are 12 inches in 1 foot, so 2 feet would be 24 inches.



Student cont'd: If she wants to make 3 bracelets, then she needs to split the 24 inches into 3 groups. I know 3×8 is 24, so 3 groups of 8 inches is 24 inches. So, Susan will use 8 inches for each bracelet.

Example: A pound of apples costs \$1.20. Roman bought a pound and a half of apples. If Roman gave the clerk a \$5.00 bill, how much change should he get back?

Student: One pound of apples would be \$1.20. Half a pound of apples would be half of \$1.20, which is 60 cents. So that's \$1.80 total. To figure out the change, I drew a number line.



Student cont'd: I counted up. I need 20 cents to jump from \$1.80 to \$2.00. And then I need 3 more dollars to get to \$5.00. So, Roman needs \$3.20 back in change.

4.MD.3

Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

In Grade 3, students were introduced to the concepts of area and perimeter. They explored area by tiling areas with square units and counting the units (**3.MD.6**). They investigated the connection between the total number of square units it takes to cover a rectangular area and the product of multiplying the side lengths (**3.MD.7**). They also solved real world problems involving finding the perimeter of polygons (**3.MD.8**).

One of the goals for Grade 4 is for students to progress to a more general understanding of area and perimeter. In Grade 3, students often worked with area representations that showed every individual square unit. In Grade 4, we would like for them to be able to determine the area of a rectangle without depending on seeing/counting individual units.

Textbooks often present “the formula” for the perimeter of a rectangle as $P = l + w + l + w$ or $P = 2(l + w)$ and the perimeter of a square as $P = 4s$ (where s is the side length). But the process for finding the perimeter is the same: we simply add up the side lengths. Because rectangles have two opposite pairs of congruent sides, an efficient way to find the perimeter might be $P = 2(l + w)$. Similarly, all four sides of a square are congruent, hence the efficient formula of $P = 4s$.

Rather than presenting students with formulas to memorize/use, it is more appropriate at this stage to ask students to come up with a “rule” or “description” for how to find the area or perimeter of a figure and then ask thoughtful questions to help students notice mathematical patterns. If students truly understand how to find the perimeter of a polygon, they don’t need to memorize multiple formulas for perimeter.

TEACHER NOTE: The language we use in discussing area at this stage is very important. It is common for teachers (and students) to say, “Area equals length times width” or “Area is base times height.” But this is not always true. The areas of triangles, circles, trapezoids, (and other shapes) is not found by the formula $A = l \times w$. But once students “learn” that phrase, it is very difficult to get them away from it. It would be more accurate to say, “The area of a *rectangle* can be found by multiplying length times width.”

For a great example of classroom tasks/discussion that allow students to explore area and perimeter in a real world context, see the teaching article

- Moyer, P. (2001, September). Using representations to explore perimeter and area. *Teaching Children Mathematics*, 52-59.

For examples of common misconceptions students make in exploring area and perimeter and tasks to elicit discussion of those misconceptions, see the teaching article

- Chappell, M. F., & Thompson, D. R. (1999, September). Perimeter or area: Which measure is it? *Mathematics Teaching in the Middle School*, 5(1), 20-23.

Cluster**Represent and interpret data.**

Vocabulary: data, line plot, length, fractions

4.MD.4

Make a line plot to display a data set of measurements in fractions of a unit ($1/2$, $1/4$, $1/8$). Solve problems involving addition and subtraction of fractions by using information presented in line plots.

For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

This Standard builds directly off of **3.MD.4** in which students use rulers to measure lengths to the nearest whole number, half, or fourth, and then construct a line plot with that data. Here, students extend that work to include measurements to the nearest eighth of a unit. This Standard is also intended to work with **4.NF.3c** and/or **4.NF.3d** in which students add and subtract fractional amounts with like denominators.

Example: Farmer Fred recently planted a batch of tomato seeds. On Monday, he measured how tall (in inches) the new plants are so far. He recorded his measurements in the line plot below.



Use Farmer Fred's line plot to answer the following questions:

- (1) How many total tomato plants did Farmer Fred measure on Monday? How did you figure that out?
- (2) How tall are the tallest tomato plants in Farmer Fred's garden?*
- (3) How tall are the shortest tomato plants in Farmer Fred's garden?***
- (4) How much taller is one of the tallest plants than one of the shortest plants?
- (5) If Farmer Fred were able to stack all of the new plants that measure $1/8$ of an inch on top of each other, how tall would they be together? You may use a ruler to help picture this.
- (6) Would the stack from (5) be taller or shorter than $1/2$ of an inch? How do you know?

* Teachers, be prepared for students to pick $1/4$ of an inch as "the tallest" because the column of x's above $1/4$ is the tallest in the display. This is incorrect but provides an opportunity to discuss how to interpret a line plot.

** Teachers, be prepared for students to pick $1/2$ of an inch as "the shortest" because the column of x's above $1/2$ is the shortest in the display. This is incorrect but, again, provides an opportunity to discuss how to interpret a line plot correctly.

Cluster**Geometric Measurement: understand concepts of angle and measure angles.**

Vocabulary: measure, point, end point, geometric shapes, ray, angle, circle, fraction, intersect, one-degree angle, protractor, decompose, addition, subtraction, unknown, obtuse, acute

4.MD.5

Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

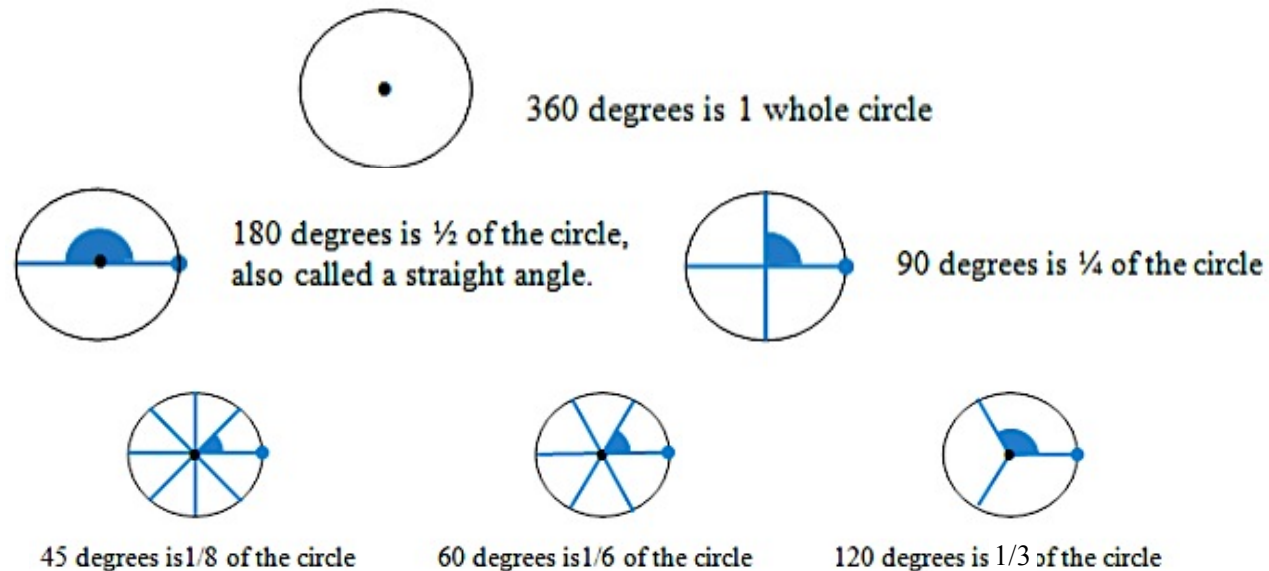
- a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1/360$ of a circle is called a "one-degree angle," and can be used to measure angles.
- b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

An angle

name	measurement
right angle	90°
straight angle	180°
acute angle	between 0 and 90°
obtuse angle	between 90° and 180°
reflex angle	between 180° and 360°

(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 23)

Just as benchmark fractions (4.NF.2) and benchmark measurements (4.MD.1) are important for students, benchmark angles are also important and useful in developing students' understanding of angle measurement. Useful benchmark angles include 180° , 90° , 45° , and 120° . Students' work in exploring fractional amounts of circles can be helpful in establishing visual references for these fraction amounts. For example,



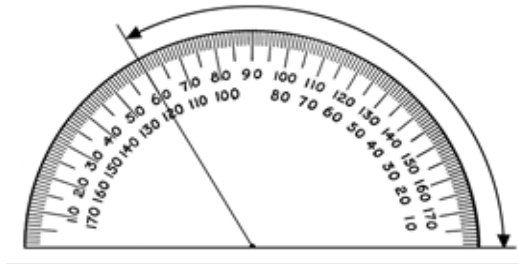
(Common Core Georgia Performance Standards Framework, Fourth Grade Mathematics Unit 7, Georgia Department of Education, p. 133, 135)

4.MD.6

Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

“Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students should decide first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90° , it is an acute angle and its measure ranges from 0° to 89° . If the angle appears to be an angle that is greater than 90° , it is an obtuse angle and its measures range from 91° to 179° . Ask questions about the appearance of the angle to help students in deciding which number to use.” (Common Core Georgia Performance Standards Framework, Fourth Grade Mathematics Unit 7, Georgia Department of Education, p. 133)

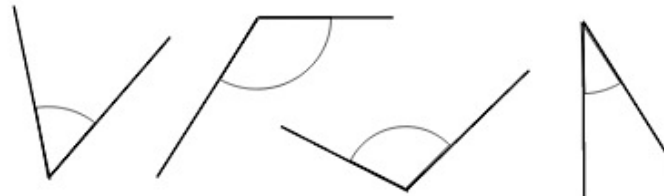
Example: Is the angle protractor 60° or 120° ?



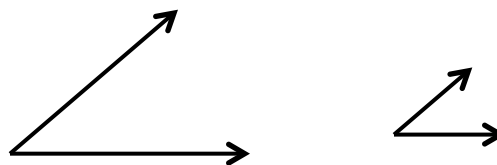
measure indicated by the protractor?
How can you tell?

Student: The angle is bigger than a right angle, which is 90° . So, the angle measure must be 120° .

It is also important for students to have opportunities to measure angles in different orientations. For example:



A common misconception is for students to think that the size of the rays is related to the size of the angle:

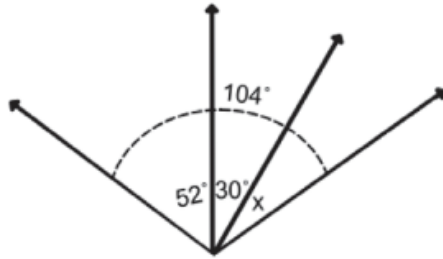


Students with this misconception will claim that the angle on the left is “bigger” than the angle on the right because the rays of the angle on the left are bigger, even though the angle measures between the rays are the same. These students lack a true understanding of what angles measure (the rotational distance between the rays). Explorations with a protractor or “tracing” the arc with a finger can help address these misconceptions.

4.MD.7

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Example: Find the missing angle using an equation.



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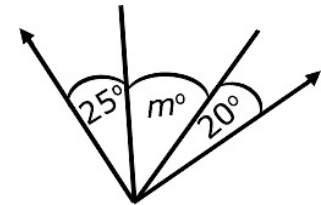
Important Terminology:

Two or more adjacent angles are said to be *complementary angles* if the sum of their angles is 90° .

Two or more adjacent angles are said to be *supplementary angles* if the sum of their angles is 180° .

Example:

If the measure between the two rays is 90 degrees, what is the value of angle m ?



<u>Student Response</u>	
$25 + 20 = 45$	<u>Check:</u>
$90 - 45 = \underline{\quad}$	$20 + 45 = 65$
$90 - 40 = 50$	$65 + 25 = \underline{\quad}$
$50 - 5 = 45.$	$65 + 20 = 85$
45°	$85 + 5 = 90$

Example:

Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30° . What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?

Student: I used a chart to help me think about it:

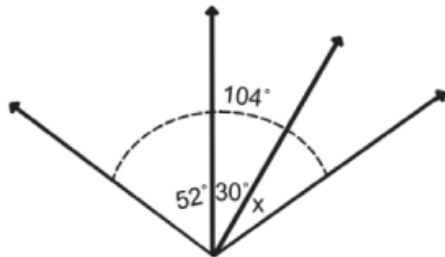
<u>clock hands</u>	<u>angle</u>	<u>total</u>
12 - 1	30°	30°
1 - 2	30°	60°
2 - 3	30°	90°
3 - 4	30°	120°

Student (cont'd): So, when the clock hands are on the 12 and the 4, the angle measure would be 120° .

4.MD.7 (cont'd)

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Example: Find the missing angle using an equation.



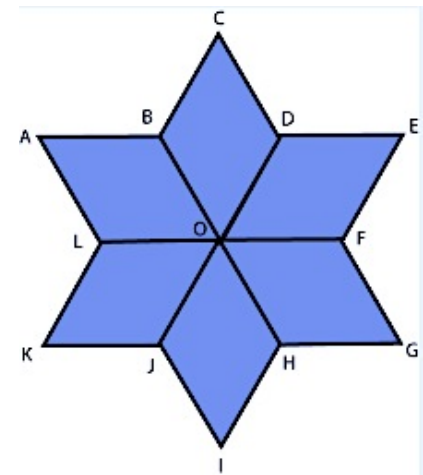
Example:

A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degree circle, how many times will the water sprinkler need to be turned?

<u>Student Response</u>	
65	$4 \times 9 = 36$
$+25$	$4 \times 90 = 360$
80	
$\underline{10}$	<i>It's already gone one 90° rotation, so it needs to go 3 more 90° rotations to make a full circle. So, you need to turn it 3 times to make the full 360°.</i>
90	

Example: Given the figure to the right, use what you know about angles to find the measure of the following angles:

- $\angle BOD =$ _____
- $\angle BOF =$ _____
- $\angle ODE =$ _____
- $\angle CDE =$ _____



(Progressions for the CCSSM, K-5 Geometric Measurement, CCSS Writing Team, June 2012, page 24)

Geometry

Cluster

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

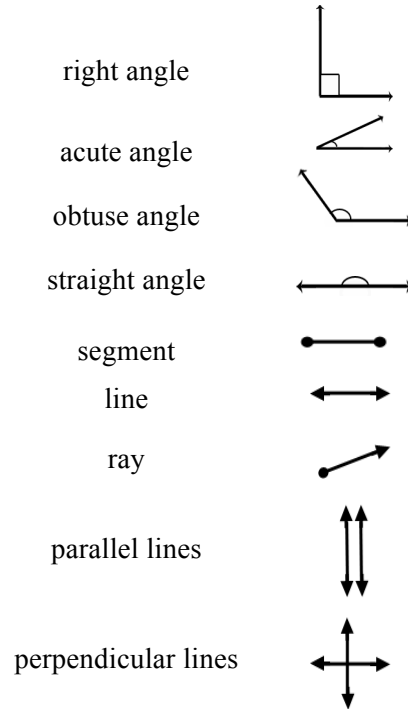
Vocabulary: classify shapes/figures, properties of shapes, point, line segment, ray angle, vertex/vertices, right angle, acute angle, obtuse angle, perpendicular, parallel, right triangle, isosceles triangle, equilateral triangle, scalene triangle, line of symmetry, symmetric figures, two-dimensional, regular, irregular, sides

From previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, trapezoid, kite

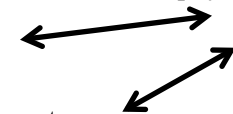
4.G.1

Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

Discussions of angles in this Standard work well with 4th Grade Measurement Standards (4.MD.5, 4.MD.6)



Students often struggle with the concept that lines extend *infinitely* in both directions. We are limited in how we can represent this; traditionally we do so by drawing arrows on both ends of a line to represent that it continues in both directions. It is a common misconception for students to look at lines that do not physically intersect (such as the ones to the right) and decide that they are parallel because “the lines don’t cross,” as they are currently drawn. Students should be encouraged to use rulers to extend the lines and to see how, if they *did* extend forever (the way that true mathematical lines do), they *would* intersect.



(continued on next page)

4.G.1 (cont'd)

Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

Students often do not realize that there may be more than one way to classify a set of shapes. For example, triangles can be classified by side length (equilateral, isosceles, scalene) or by angle measure (equiangular, right, acute, obtuse). These descriptors can be used together. For example, you could draw an isosceles right triangle or a scalene right triangle.

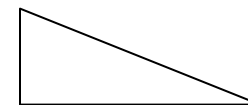
When classifying triangles by angle measures, traditionally triangles are named according to the largest angle in the triangle. So, even though a triangle with an angle of 90 degrees will have two other angles measuring less than 90 degrees, we would not call it an acute triangle. We would call it a right triangle because its largest angle measures 90 degrees.

Just as certain quadrilaterals are related (i.e., a square is a special type of rectangle because it has 4 right angles, 2 opposite pairs of parallel sides, and all congruent sides), there is a similar relationship among certain triangles. An isosceles triangle has *at least two* congruent sides. An equilateral triangle has all three congruent sides. **So an equilateral triangle is, in fact, a special type of isosceles triangle.**

Thought-provoking tasks can help students make sense of geometric shapes in meaningful & interesting ways:

Example: Is it possible to draw a scalene right triangle? If so, draw it and explain how your picture fits the question. If not, explain why it is impossible.

Student: Scalene means that all three of the sides have different lengths. A right triangle means that the biggest angle is 90 degrees. So, here is my picture:

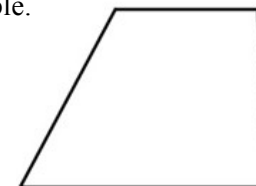


Example: Is it possible to draw an equiangular right triangle? If so, draw it and explain how your picture fits the question. If not, explain why it is impossible.

Student: Equiangular means all three sides are the same length. The sum of the interior angles of a triangle is 180 degrees. You can't have three 90 degree angles in there; you'll run out of room. This one isn't possible.

Example: Is it possible to draw a scalene quadrilateral with exactly one pair of parallel sides? If so, draw it and explain how your picture fits the question. If not, explain why it is impossible.

Student: I drew this. The top and bottom sides are parallel, but none of the sides are the same length. So, I say this fits the description.

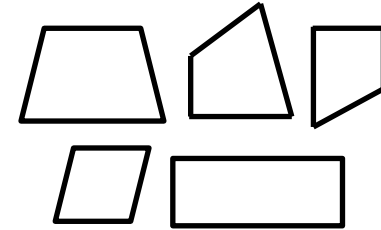


4.G.2

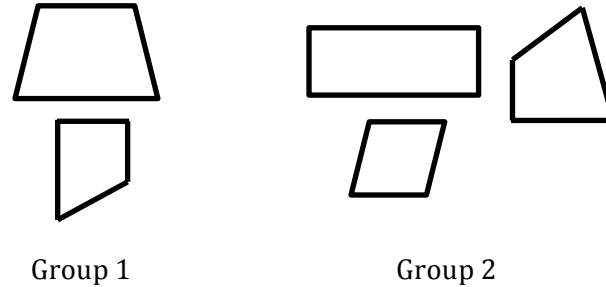
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

This standard calls for students to sort objects based on parallelism, perpendicularity and angle types. Students can build on their work with comparing and classifying shapes in Grade 3 (3.G.1) and extend the “rules” to include these properties. Students should have extensive experiences in sorting two-dimensional figures into groups based on the figures’ defining attributes. Having students create and use a “rule” to sort the shapes into groups can be a rich way to help them see different attributes in the shapes.

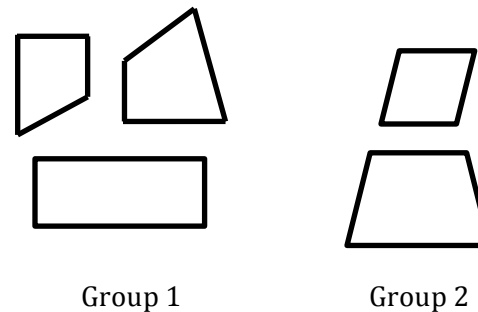
Example: Make a rule to sort these shapes into two different groups.



Student A: I used the rule, “Shapes that have exactly one pair of parallel sides go in Group 1. Shapes that do not have exactly one pair of parallel sides go in Group 2.” These are my groups:



Student B: I used the rule, “Shapes that have 1 or more right angles go in Group 1. Shapes that don’t have any right angles go in Group 2.” These are my groups:

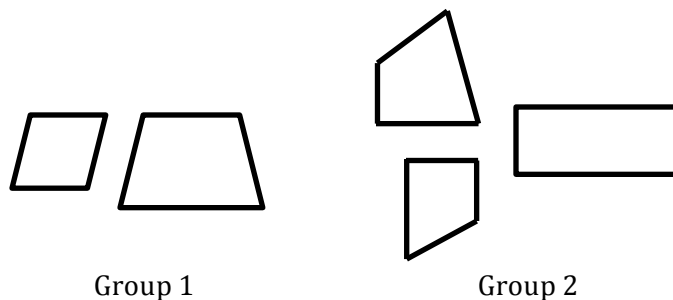


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4.G.2 (cont'd)

Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Student C: I used the rule, “Shapes that don’t have any perpendicular sides go in Group 1. Shapes that have at least 1 pair of perpendicular sides go in Group 2.” These are my groups:



TEACHER NOTE: In the U.S., the term “trapezoid” may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. With this definition, parallelograms, rectangles, squares, and rhombi fit that definition and can thus be considered as *types of trapezoids*. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. With this definition, parallelograms and their subgroups do not fit the definition and thus are not considered to be types of trapezoids. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012.)

A **kite** is a quadrilateral with two pairs of adjacent congruent sides and whose diagonals form right angles. A **right kite** has at least one angle that measures 90 degrees.

4.G.3

Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Students need experiences with figures that are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cutout figures can help students develop initial understandings that a figure folded along a line of symmetry will have matching parts on either side. They can also determine whether a figure has one or more lines of symmetry.

Students should use reflectors or MIRAs to look for and create lines of symmetry in two-dimensional figures. They can also use these tools to “finish” a figure, given part of a figure and a line of symmetry to reflect across. At this young age, students often struggle to draw figures and lines accurately, which can frustrate them or distract them from the deeper mathematical concepts and relationships they need to focus on. Using reflectors and MIRAs can help students draw figures accurately while developing their conceptual understandings of lines of symmetry and symmetrical relationships.

This standard only includes line symmetry, not rotational symmetry.

Table 1: Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ (K)	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ (1 st)	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ One-Step Problem (2 nd)
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ (K)	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ (1 st)	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ One-Step Problem (2 nd)
	Total Unknown	Addend Unknown	Both Addends Unknown
Put Together/Take Apart	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ (K)	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ or $5 - 3 = ?$ (1 st)	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$ (K)
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (1 st)	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? One-Step Problem (1 st)	(Version with “more”): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ or $? + 3 = 5$ One-Step Problem (2 nd)
	(“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ or $5 - 2 = ?$ (1 st)	(Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ or $3 + 2 = ?$ One-Step Problem (2 nd)	(Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$ One-Step Problem (1 st)

K: Problem types to be mastered by the end of the Kindergarten year. **1st**: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types. **2nd**: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Table 2: Common Multiplication and Division Situations¹

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays ² , Area ³	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

¹The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

²The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

³Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

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